CFA LEVEL II FIXED INCOME REVIEW
JULY 2023

KEY POINTS, PRACTICE PROBLEMS & SOLUTIONS

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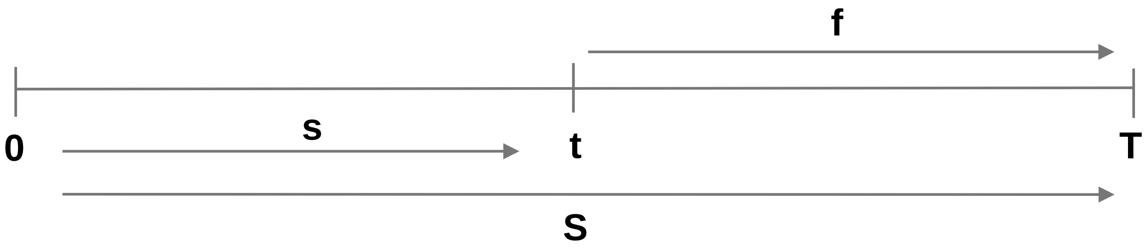
LM1 Describe Relationships among spot rates and forward rates

- Spot Rates – zero coupon rates observed in today's yield curve.
- Forward Rates – implied by the spot curve for periods in between spot rates.

$$(1 + S)^T = (1 + s)^t * (1 + f)^{(T - t)}$$



Year	1	2	3	4	5	6	7	8	9	10
Spot Rate Curve	1.00%	1.50%	2.00%	2.50%	3.00%	3.50%	4.00%	4.50%	5.00%	5.50%
1yr forwards		2.00%	3.01%	4.01%	5.02%	6.04%	7.05%	8.07%	9.09%	10.11%

Example: $0.0604 = \dfrac{(1+.035)^{6}}{(1+.030)^{5}} - 1$

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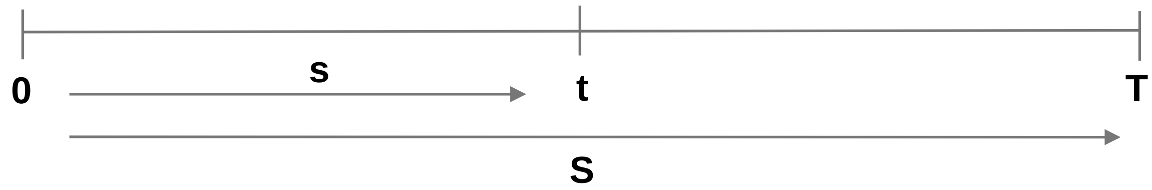
LM1

Describe the forward pricing and forward rate models and calculate forward and spot prices using these models.



Year	1	2	3	4	5	6	7	8	9	10
Spot Rate Curve	1.00%	1.50%	2.00%	2.50%	3.00%	3.50%	4.00%	4.50%	5.00%	5.50%
P	0.9901	0.9707	0.9423	0.9060	0.8626	0.8135	0.7599	0.7032	0.6446	0.5854
1 yr forward rates	-	2.00%	3.01%	4.01%	5.02%	6.04%	7.05%	8.07%	9.09%	10.11%
1 yr forward Ps	-	0.9804	0.9708	0.9614	0.9522	0.9431	0.9341	0.9253	0.9167	0.9082

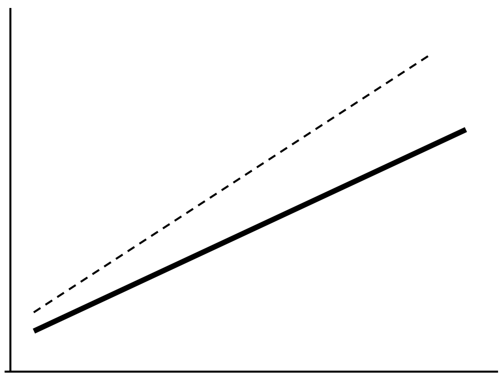
F(1,2) = P(2) / P(1) = .9707 /.9901 = .9804

P(5) = P(1) * F(1,2) * F(2,3) * F(3,4) * F(4,5)

LM1 Describe yield to maturity, expected returns on bonds, and the shape of the yield curve.

- Yield to Maturity – weighted average of all the discount rates used to price the cash flows of a bond.
- $$P = \frac{C}{(1+y)} + \frac{C}{(1+y)^2} + \ldots \frac{(C + \text{Principal})}{(1+y)^n}$$
- YTM does not equal the expected return of the bond unless the bond is held to maturity, all coupon and principal payments are made, and all coupons are reinvested at the original YTM.

Upward Sloping Curve

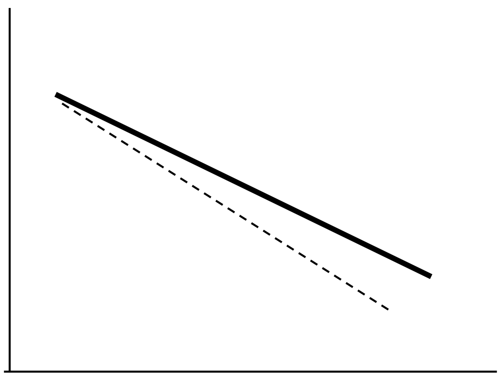


Forward rates > spot rates

Flat Curve

Forward rates = spot rates

Inverted Curve



Forward rates < spot rates

LM1 **Describe how zero-coupon rates (spot rates) may be obtained form the par curve by bootstrapping.**

- Knowing that a bond with a coupon equal to the par rate has a price of 100 (the sum of its discounted cash flows), we can solve for the forward rates needed to discount each coupon as well as the final principal.

- To illustrate this concept, assume all bonds pay 1 annual coupon at the end of the period. This makes the 1-year par rate equal to the 1-year zero rate.

Year	Par rates	Zero rates
1	1.00%	1.000%
2	1.50%	1.495%
3	2.00%	2.004%
4	2.50%	2.532%

$$1 = \frac{1^{st}\text{ yr cpn}}{(1 + ZC(0,1))} + \frac{2^{nd}\text{ yr cpn + principal}}{(1 + ZC(1,2))^2}$$

$$1 = \frac{1^{st}\text{ yr cpn}}{(1 + ZC(0,1))} + \frac{2^{nd}\text{ yr cpn}}{(1 + ZC(1,2))^2} + \frac{3^{rd}\text{ yr cpn + principal}}{(1 + ZC(1,3))^3}$$

And so on…

Practice Problem 1

Use the following spot rate curve to answer this question:

Maturity	1	2	3
Spot rates	5%	5.5%	6%

The 1-year forward rate in one year [$f(1,1)$] and the 1-year forward rate in two years [$f(2,1)$] is *closest* to:

	$f(1,1)$	$f(2,1)$
A)	6%	7%
B)	5.25%	5.75%
C)	4%	4.89%

Practice Problem 1

Solve for f(1,1):

$(1 + S_2)^2 = (1+S_1) * (1 + f(1,1))$

$(1 + 0.055)^2 = (1+0.05) * (1 + f(1,1))$

$1.113025 = 1.05 * (1 + f(1,1))$

$$f(1,1) = \frac{1.113025}{1.05} - 1 = 0.060024 \sim 6\%$$

You could then use the same methodology to solve for f(2,1) but by looking at the answers available you don't really have to.

The correct answer is A.

Practice Problem 2

The following are some of the current par rates:

Year	Par rate
1	5.00%
2	6.00%
3	7.00%

Using bootstrapping, the 3-year spot rate is closest to:

- A) 6.67%
- B) 6.93%
- C) 7.09%

Practice Problem 2

First solve for the 2-year spot rate.

A 2-year par bond has a price of 100 and its YTM is the average of the spot rates used to discount each coupon received plus the payback of principal.

S_1 = 5.00% given

For the 2-year par bond,

$$100 = \frac{6.00}{(1.05)} + \frac{106}{(1+S_2)^2}$$

$$94.29 = \frac{106}{(1+S_2)^2}$$

$(1+S_2)^2$ = 106/95.24 = 1.1242

$(1+S_2)$ = 1.0603

S_2 = 6.03%

Then use the 2-year spot rate to plug into the same equation for a 3-year par bond.

For the 3-year par bond,

$$100 = \frac{7.00}{(1.05)} + \frac{7.00}{(1.0603)^2} + \frac{107.00}{(1+S_3)^3}$$

$$87.11 = \frac{107.00}{(1+S_3)^3}$$

$(1+S_3)^3$ = 107/87.11 = 1.2283

$(1+S_3)$ = 1.0709 or S_3 = 7.09%

The correct answer is C.

Practice Problem 3

If the 2-year spot rate is 4% and 1-year spot rate is 7%, the one year forward rate one year from now is *closest* to:

- **A)** 3%
- **B)** 1%
- **C)** 2%

Practice Problem 3

Solve for f(1,1):

Long way:

$(1 + S_2)^2 = (1+S_1) * (1 + f(1,1))$

$(1 + 0.04)^2 = (1+0.07) * (1 + f(1,1))$

$1.0816 = 1.07 * (1 + f(1,1))$

$$f(1,1) = \frac{1.0816}{1.07} - 1 = 1.01084 \sim 1\%$$

Short way:

The correct forward rate means that the rollover strategy will have the same total return as the full holding period strategy.

Horizon is 2 years
Total return of 2-year bond @ 4% per year = 8
Total return of 1-year bond @ 7% per year = 7

The total return on a fairly priced 1-year bond 1 year from now has to be 8 – 7 = 1.

The correct answer is B.

Practice Problem 4

Use the following spot rate curve to answer this question:

Maturity	1	2	3
Spot rates	5%	5.5%	6%

The price of a 1-year $1 par, zero-coupon bond to be issued in two years is *closest* to:

- **A)** $0.8396
- **B)** $0.9345
- **C)** $0.9434

Practice Problem 4

Solve for f(2,1) so that we can get the price of the bond:

$(1 + S_3)^3 = (1+S_2)^2 * (1 + f(1,1))$

$(1 + 0.06)^3 = (1+0.055)^2 * (1 + f(1,1))$

$1.113025 = 1.113025 * (1 + f(1,1))$

$$f(1,1) = \frac{1.191016}{1.113025} - 1 = .070071$$

$$P = \frac{1}{(1 + .070071)}$$ = 0.9345 since we are told par is $1.

The correct answer is B.

Practice Problem 5

Don McGuire, fixed income specialist at MCB bank makes the following statement: "In the very short-term, the expected rate of return from investing in any bond, including risky bonds, is the risk-free rate of return".

McGuire's statement is *most* consistent with:

- **A)** local expectations theory.
- **B)** unbiased expectations theory.
- **C)** liquidity preference theory.

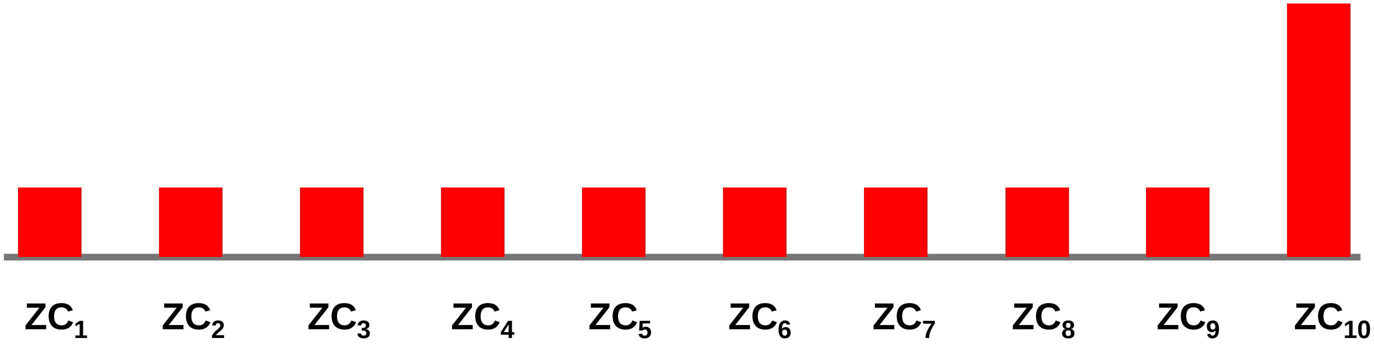
Practice Problem 5

Local Expectations Theory believes that forward rates are an unbiased predictor of the future spot rates. If true, then all bonds will converge to their respective forward prices. Therefore, they will all have the same return regardless of maturity.

The correct answer is A.

Explain what is meant by arbitrage-free valuation of a fixed income instrument.

- Securities with the same cash flows have the same price.
- Any coupon bond can be thought of as a package of zero coupons bonds
- Discount cash flows using the appropriate spot (zero) rate.



- Value additivity must hold in quoted prices (generally based on yield to maturity) or an arbitrage opportunity will exist.

LM2 Calculate the arbitrage-free value of an option-free, fixed rate coupon bond.

- Given par (coupon) rates based on yield to maturity.
- Bootstrap to zero coupon rates to get discount factors.
- Value the bond's cash flows vs. the price quoted.
- See if there is a difference.

Maturity	Par (Coupon) Curve	Zero Rates
1	5.00%	5.000%
2	5.97%	6.000%
3	6.91%	7.000%
4	8.00%	8.223%
5	8.50%	8.790%

$$\frac{C}{(1 + YTM)^1} + \frac{C}{(1 + YTM)^2} + \ldots \frac{C + P}{(1 + YTM)^T} = \frac{C}{(1 + ZC_1)^1} + \frac{C}{(1 + ZC_2)^2} + \ldots \frac{C + P}{(1 + ZC_n)^n}$$

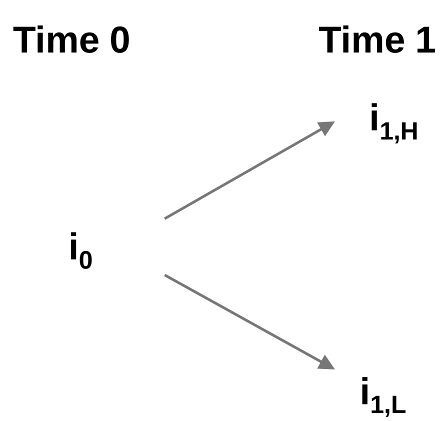
Describe a binomial interest rate tree framework.

- Discounting with spot rates is fine for option-free bonds but for bonds with embedded options (e.g. calls/puts) we need to develop a framework which allows for different interest rate paths from today to maturity.

- Valuing a bond with embedded options will depend on the which path is followed which determines whether the option is exercised and in so doing will change the cash flows of the bond.

- Our model looks like a tree or a lattice as we now allow some volatility around each forward rate from our spot curve to generate more nodes on the path.

- Start with a given short rate and volatility σ.
  - In each time step the short rate can move up or down a given amount (σ) with a 50% probability
  - Interest rates follow a lognormal random walk which posits the relationship below and ensures that nodes combine to make a tree.
  - Each node will have a discount rate associated with it.



$$i_{1,H} = i_{1,L}\, e^{2\sigma}$$

Know this! If this condition is not met the tree will fail to combine

LM2

Describe the backward induction valuation methodology and calculate the value of a fixed-income instrument.

- Backward Induction: Start at end of the branches and work backwards. The terminal value at maturity is par plus the last coupon received.
- Each node's bond value is the present value of the next period's full market value (principal + coupon) in both the +1σ and – 1σ scenarios multiplied by the probability of that path being followed (50%).

$$V = \frac{.5\,(V_H + C) + .5\,(V_L + C)}{1 + r^*}$$

r* = 1 period forward rate

You always start at maturity and work your way back one node at a time.

The path of interest rates (forward rates) in the tree will be determined by the yield volatility assumption.

Practice Problem 1

Jorgen Welsher, CFA obtains the following quotes for zero coupon government bonds all with a par value of $100.

Type of Price	Delivery (years)	Maturity (years)	Price
Spot	0	3	$91.51
Forward	2	3	$94.55
Spot	0	2	$92.45

Welsher can earn arbitrage profits by:

- **A)** buying the 2-year bond in the spot market, going long the forward contract and selling the 3-year bond in the spot market.
- **B)** buying the 2-year bond in the spot market, going short the forward contract and selling the 3-year bond in the spot market.
- **C)** selling the 2-year bond in the spot market, going short the forward contract and buying the 3-year bond in the spot market.

Practice Problem 1

The horizon return is 3 years.

Solve for the price/rate of the 1-year bond starting 2 years from now and compare it to the forward contract being offered to see whether a buy and hold or a rollover strategy is better.

We can calculate the 3-year spot = 3% and the 2-year spot = 4%

Recall that F(2,1) = P3 / P2 = 91.51 / 92.45 = .9898 or $98.98 for $100 par.

This is the no arbitrage price and implies a rate of 1.03%. (check vs. spot rates)

Since the forward is offered at $94.55 it is too cheap – its implied rate is 5.76%!

For a 3-year horizon which offers a higher yield?

2-year spot + forward - *OR* - 3-year bond

$(1 + 0.04)^2 + (1 + 0.0576) > (1 + 0.03)^3$ so arbitrage exists.

To take advantage buy the cheap package and sell the fair package.

The correct answer is A.

Practice Problem 2

Sam Roit, CFA, has collected the following information on the par rate curve, spot rates, and forward rates to generate a binomial interest rate tree consistent with this data.

Maturity	*Par Rate*	*Spot Rate*
1	5%	5.000%
2	6%	6.030%
3	7%	7.097%

The binomial tree generated is shown below (one year forward rates) assuming a volatility level of 10%:

0	1	2
5%	7.7099%	C
	A	9.2625%
		B

Riot also generated another tree using the same spot rates but this time assuming a volatility level of 20% as shown below:

0	1	2
5%	8.9480%	13.8180%
	5.9980%	9.2625%
		6.2088%

Is the binomial tree using the 20% volatility assumption calibrated properly?

- **A)** The tree is calibrated properly.
- **B)** The tree is not calibrated properly because adjacent nodes are not appropriate standard deviations apart.
- **C)** The tree is not calibrated properly because it is not consistent with market prices.

Practice Problem 2

Test the tree determined by 20% volatility to determine the present value of an option free bond and make sure it is equal to the observed market price.

The tree is not calibrated properly - it does not value 3-year 7% bond at par (i.e., the market price):

$$V_{2,UU} = \frac{107}{(1.13818)} = \$94.01$$

$$V_{2,UL} = \frac{107}{(1.092625)} = \$97.93$$

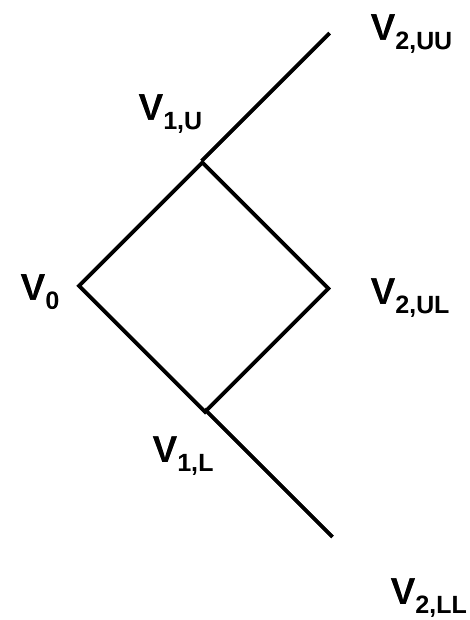
$$V_{2,LL} = \frac{107}{1.062088} = \$100.74$$

$$V_{1,U} = \frac{1}{1.08948} \times \left[\frac{94.01 \ \ +97.93}{2} + 7\right] = \$94.51$$

$$V_{1,L} = \frac{1}{1.05998} \times \left[\frac{97.93 + 100.74}{2} + 7\right] = \$100.31$$

$$V_0 = \frac{1}{1.05} \times \left[\frac{94.51 + 100.32}{2} + 7\right] = \$99.44$$

The adjacent nodes in the binomial tree for any nodal period are all two standard deviations apart.



The correct answer is C.

LM3

Describe fixed income securities with embedded options.

- Options are usually contingent on interest rates.
- These are rights that enable either the holders or the issuers to take advantage of interest rate movements.
- Typical embedded option bonds include:
  - Callable bond. Issuer has the right to call the bond from the holder if rates fall. Price is often set at par but can be at a spread to treasuries ("make whole call").
  - Putable bond. Holder has the right to put the bond back to the issuer if rates rise.
  - Sinking fund bond. Issuer is required to set aside funds to retire the bonds but may also include a schedule of partial calls.
  - Convertible bond. Holder has a right to purchase equity in the issuer.

LM3

Explain the relationships between the value of a callable or putable bond, the underlying option-free (straight) bond, and the embedded option.

- The value of a bond with embedded options is the sum of the arbitrage free value of the straight bond plus the arbitrage free value of the embedded options.
- Value of callable bond = value of option free bond – value of issuer call option.

  - using example: call option = 104.643 – 102.899 = 1.744

  - *Callable bonds have lower prices and higher yields than straight bonds.*
- Value of putable bond = value of option free bond + value of holder put option.

  - Using example: put option = 105.327 – 104.643 = 0.684

  - *Putable bonds have higher prices and lower yields than straight bonds.*

LM3 Calculate the value of a callable or putable bond from an interest rate tree.

- Recall the basic backward induction calculation:

$$V = \frac{.5\,(V_H + C) + .5\,(V_L + C)}{1 + r^*}$$

- For a callable bond replace every node ***beyond the lockout period*** that has a price ***above*** the call price, with the call price and then re-calculate.

- In the case of a putable bond, replace every node ***beyond the lockout period*** that has a price ***below*** the put price, with the put price and then re-calculate.

- While call and put prices are often par (100) they do not have to be. Be sure to read the question carefully to understand the call/put price and the relevant lockout period.

Practice Problem 1

Which of the following is equal to the value of the putable bond? The putable bond value is equal to the:

- **A)** option-free bond value plus the value of the put option.
- **B)** callable bond plus the value of the put option.
- **C)** option-free bond value minus the value of the put option.

Practice Problem 1

Recall:

- Callable bond. Issuer has the right to call the bond from the holder if rates fall.
  (you sold the option to the issuer)

- Putable bond. Holder has the right to put the bond back to the issuer if rates rise.
  (you bought the option from the issuer)

Value of callable bond = value of option free bond – value of issuer call option.

Value of putable bond = value of option free bond + value of holder put option.

The correct answer is A.

Practice Problem 2

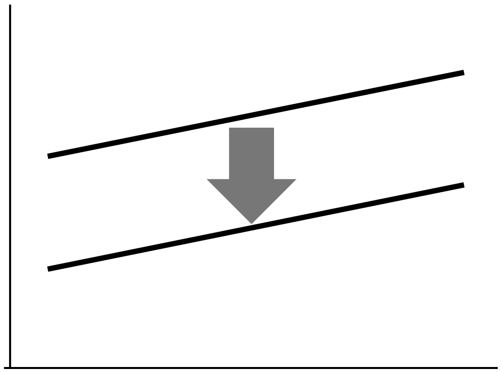
Bill Moxley, CFA is evaluating three bonds for inclusion in fixed income portfolio for one of his pension fund clients. All three bonds have a coupon rate of 3%, maturity of five years and are generally identical in every respect except that bond A is an option-free bond, bond B is callable in two years and bond C is putable in two years. The yield curve is currently flat.

If the yield curve is expected to have a parallel downward shift, the bond with the highest price appreciation is *least likely* to be:

- **A)** Bond B
- **B)** Bond A
- **C)** Bond C

Practice Problem 2

Duration Shift (Parallel)



In a downward shift the bond with the highest duration will perform the best.

The callable bond's price grows at a decreasing rate as yields fall and eventually stops appreciating as the price hits the call price.

The correct answer is A.

Practice Problem 3

Using the following tree of semiannual interest rates what is the value of a putable bond that has one year remaining to maturity, a put price of 99, coupons paid semiannually with payments based on a 5% annual rate of interest?

	7.59%
6.35%	
	5.33%

- A) 99.00.
- B) 98.75.
- C) 97.92.

Practice Problem 3

The putable bond price tree is as follows:

		100.00
	A → 99.00	
99.00		100.00
	99.84	
		100.00

As an example, the price at node A is obtained as follows:

Price_A = max[(prob × (P_{up} + coupon / 2) + prob × (P_{down} + (coupon / 2)) / (1 + (rate / 2)), put price] = max[(0.5 × (100 + 2.5) + 0.5 × (100 + 2.5)) / (1 + (0.0759 / 2)) ,99] = 99.00. The bond values at the other nodes are obtained in the same way.

The calculated price at node 0 =

[0.5(99.00 + 2.5) + 0.5(99.84 + 2.5)] / (1 + (0.0635 / 2)) = $98.78 but since the put price is $99 the price of the bond will not go below $99.

The correct answer is A.

Practice Problem 4

Relative to the binomial model, Monte Carlo method is *most likely*:

- **A)** more suitable when valuing securities whose cash flows are interest rate path dependent.
- **B)** more flexible as it does not need a volatility estimate.
- **C)** less flexible in forcing interest rates to mean revert.

Practice Problem 4

Monte Carlo method does not require that cash flows of a security are path dependent and hence is suitable alternative to the binomial model to value securities such as mortgage backed securities whose cash flows are path dependent. The model generating interest rates paths in a Monte Carlo simulation is based on an assumed level of volatility (i.e., model needs a volatility input). The model generating interest rates in a Monte Carlo simulation can incorporate bounds for interest rates to force mean reversion of rates. Such bounded optimization is not possible in a binomial model.

The correct answer is A.

Practice Problem 5

For a convertible bond, which of the following is *least* accurate?

- **A)** A convertible bond may be putable.
- **B)** The conversion ratio times the price per share of common stock is a lower limit on the bond's price.
- **C)** The issuer can decide when to convert the bonds to stock.

Practice Problem 5

A convertible bond is a security that can be converted into common stock at the option of the *investor*.

Conversion ratio = # of shares of common stock for each unit of bond (1,000 par bond = 25.32 shares) = 25.32 conversion ratio.

There is no floor on the price of a convertible bond.

Convertible bonds can be callable or putable too.

The correct answer is C.

Relevant Definitions

Credit Spread: The difference in yields to maturity on a corporate bond and a government bond with the same maturity. The extra compensation required by investors for bearing the default risk of the issuer (borrower).

Expected Exposure: The projected amount of money the investor could lose if an event of default occurs *including all expected coupon payments* but not factoring in any possible recovery.

Default: When the borrower fails to satisfy the terms of its debt obligation with respect to the timely payment of interest and repayment of principal.

Recovery rate: The percentage of the loss recovered in default. This is often the number that is given, i.e. the estimated value of property, plant, and equipment. *Here it is assumed that this applies to interest as well and that it is instantaneous.*

Loss Severity: The percentage lost in a default, or (1– Recovery Rate).

Loss Severity + Recovery Rate = 100%

Loss given default: The amount of the remaining coupon and principal payments lost in the event of default. Most often calculated as (1 - Recovery rate) x Expected Exposure.

What you are left with if default occurs is: Expected Exposure – Loss Given Default

Probability of Default Types

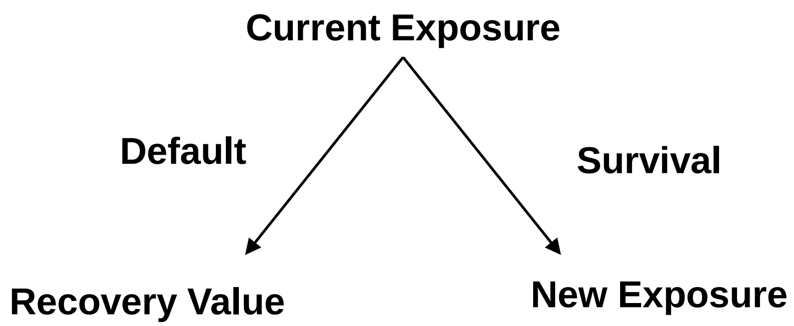
Probability of default: The probability that the bond will default before maturity.

Actual Probability. Historical data captured by rating agencies counting the number of defaults. Often segregated by bond type, rating, industry, etc.

Risk Neutral Probability. Discounts the expected value of payoffs using the risk-free rate to match the bond's current price. Denoted as P* in the reading – solve the equation below to determine its value.

$$\text{Current Bond Price} = \frac{(\text{Exposure} \times (1 - P^*)) + ((\text{Exposure} - \text{Loss Given Default}) \times P^*)}{1 + \text{Risk Free Rate}}$$

In each time period the bond either defaults or continues towards maturity.



Credit Valuation Adjustment

Credit Valuation Adjustment (CVA): The present value of the expected credit loss over the life of the bond.

Example:	6% Coupon 3 year bond with a 5% POD									
			Risk Free Rate	Recovery Rate		Prob of Default				
			2.50%	40%		5.00%				
(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)
End of Year	Coupon Exposure	Principal Exposure	Total Exposure	Recovery	Loss Given Default	Probability of Default	Probability of Survival	Expected Loss	Risk Free Discount Factor	PV of Expected Loss
	PV of all coupons left	*PV of principal*	*(B) + (C)*	*Recovery rate x (D)*	*(D) - €*	*Prob of default x (H)*	*1 - Prob of default*	*(G) x (F)*	*1 / (1 + R)^N*	*(I) x (J)*
0										-
1	17.5645	95.1814	112.7460	45.0984	67.6476	5.0000%	95.0000%	3.3824	0.9756	3.2999
2	11.8537	97.5610	109.4146	43.7659	65.6488	4.7500%	90.2500%	3.1183	0.9518	2.9681
3	6.0000	100.0000	106.0000	42.4000	63.6000	4.5125%	85.7375%	2.8700	0.9286	2.6650
					Cumulative =>	14.2625%			CVA =	8.9330

Risk Free Bond				
End of Year	Coupon	Principal	Risk Free Discount Factor	PV
0				
1	6.0000		0.9756	5.8537
2	6.0000		0.9518	5.7109
3	6.0000	100	0.9286	98.4315
		Total =>		109.9961

Fair Value of Risky bond = Risk Free bond – CVA

= 109.9961 – 8.9330

= 101.0631

Credit Ratings

- Used for large companies, sovereigns, asset backed securities, etc.
- The 3 major rating agencies are Standard and Poor's, Moody's, and Fitch although others exist as well.
- Ranks the credit risk of a company but does not provide a direct estimate of default probability.
- Ratings are given as a letter grade based on the agency's evaluation of ability to repay the debt.

Letter Grade	Grade	Capacity to Repay	Representative Companies
AAA	Investment	Extremely strong	Microsoft, Johnson & Johnson
AA+, AA, AA-	Investment	Very strong	Apple, Chevron, Walmart
A+, A, A-	Investment	Strong	IBM, Goldman Sachs, Disney
BBB+, BBB, BBB-	Investment	Adequate	Boeing, AT&T, Netflix, GE
BB+, BB	Speculative	Faces major future uncertainties	Ford Motor, United Airlines
B	Speculative	Faces major uncertainties	Dish Networks, Carnival Corp
CCC	Speculative	Currently vulnerable	Rite Aid
CC	Speculative	Currently highly vulnerable	
C	Speculative	Has filed bankruptcy petition	
D	Speculative	In default	Pacific Gas & Electric

Practice Problem 1 - 8

Lena Liecken is a senior bond analyst at Taurus Investment Management. Kristel Kreming, a junior analyst, works for Liecken in helping conduct fixed-income research for the firm's portfolio managers. Liecken and Kreming meet to discuss several bond positions held in the firm's portfolios.

Bonds I and II both have a maturity of one year, an annual coupon rate of 5%, and a market price equal to par value. The risk-free rate is 3%. Historical default experiences of bonds comparable to Bonds I and II are presented below:

Bond	Recovery Rate	Percentage of Bonds That Survive and Make Full Payment
I	40%	98%
II	35%	99%

Bond III is a zero-coupon bond with three years to maturity. Liecken evaluates similar bonds and estimates a recovery rate of 38% and a risk-neutral default probability of 2%, assuming conditional probabilities of default. Kreming creates Exhibit 2 to compute Bond III's credit valuation adjustment. She assumes a flat yield curve at 3%, with exposure, recovery, and loss given default values expressed per 100 of par value.

Date	Exposure	Recovery	Loss Given Default	Probability of Default	Probability of Survival	Expected Loss	Present Value of Expected Loss
0							
1	94.2596	35.8186	58.4410	2.0000%	98.0000%	1.1688	1.1348
2	97.0874	36.8932	60.1942	1.9600%	96.0400%	1.1798	1.1121
3	100.0000	38.0000	62.0000	1.9208%	94.1192%	1.1909	1.0898
Sum				**5.8808%**		**3.5395**	**3.3367**

Practice Problems 1 - 8

Bond IV is an AA rated bond that matures in five years, has a coupon rate of 6%, and a modified duration of 4.2. Liecken is concerned about whether this bond will be downgraded to an A rating, but she does not expect the bond to default during the next year. Kreming constructs a partial transition matrix, which is presented in Exhibit 3, and suggests using a model to predict the rating change of Bond IV using leverage ratios, return on assets, and macroeconomic variables.

Partial One-Year Corporate Transition Matrix (entries in %)

From/To	**AAA**	**AA**	**A**
AAA	92.00	6.00	1.00
AA	2.00	89.00	8.00
A	0.05	1.00	85.00
Credit Spread (%)	0.50	1.00	1.75

Kreming calculates the risk-neutral probabilities, compares them with the actual default probabilities of bonds evaluated over the past 10 years, and observes that the actual and risk-neutral probabilities differ. She makes two observations regarding the comparison of these probabilities:

Observation 1: Actual default probabilities include the default risk premium associated with the uncertainty in the timing of the possible default loss.

Observation 2: The observed spread over the yield on a risk-free bond in practice includes liquidity and tax considerations, in addition to credit risk.

Practice Problem 1

1. The expected exposure to default loss for Bond I is:

   A. less than the expected exposure for Bond II.

   B. the same as the expected exposure for Bond II.

   C. greater than the expected exposure for Bond II.

Practice Problem 1

The expected exposure is the projected amount of money that an investor *could* lose if an event of default occurs, before factoring in possible recovery.

Since both bonds pay the same coupon and are quoted at the same price (par or 100) the expected exposure for both Bond I and Bond II is 100 + 5 = 105.

The correct answer is B.

Practice Problem 2

2. Based on Exhibit 1, the loss given default for Bond II is:

   A. less than that for Bond I.

   B. the same as that for Bond I.

   C. greater than that for Bond I.

Practice Problem 2

The loss given default is the expected exposure × (1 - recovery rate). It assumes default happens.

The loss given default for Bond I is 105 × (1 – 0.40) = 63.00.

The loss given default for Bond II is 105 × (1 – 0.35) = 68.25.

Higher exposure ➔ higher loss
Lower recovery rate ➔ higher loss

The correct answer is C.

Practice Problem 3

3. Based on Exhibit 1, the expected future value of Bond I at maturity is closest to:

   A. 98.80.

   B. 103.74.

   C. 105.00.

Practice Problem 3

The expected future value of the bond is the weighted average of the no-default and default amounts.

In the event of no default, the investor is expected to receive the full 105.

In the event of a default, the investor is expected to receive $105 \times 0.40 = 42$.

Weighting the 2 scenarios we get $(105 \times 0.98) + (42 \times 0.02) = 103.74$.

The correct answer is B.

Practice Problem 4

4. Based on Exhibit 1, the risk-neutral default probability for Bond I is closest to:

A. 2.000%.

B. 3.175%.

C. 4.762%.

Practice Problem 4

The risk-neutral default probability, P*, is calculated using the current price. This is the market's expectation, not the individual analyst's.

As shown in the previous problem, the expected payoff at maturity with no default is 105, and the expected payoff at maturity in the event of a default is 42.

Discounting these payoffs by the risk-free rate of interest (0.03) gets the price:

$$100 = \frac{[105 \times (1 - P^*)] + (42 \times P^*)}{1.03}$$

Solving for P* gives 0.031746, or 3.1746%.

The correct answer is B.

Practice Problem 5

5. Based on Exhibit 2, the credit valuation adjustment for Bond III is closest to:

   A. 3.3367.

   B. 3.5395.

   C. 5.8808.

Practice Problem 5

The CVA is the sum of the present value of expected losses on the bond, which from Exhibit 2 is 3.3367. No math required.

The correct answer is A.

Practice Problem 6

6. Based on Exhibit 3, if Bond IV’s credit rating changes during the next year to an A rating, its expected price change would be closest to:

A. –8.00%.

B. –7.35%.

C. –3.15%.

Practice Problem 6

The expected percentage price change is the product of the negative of the modified duration and the difference between the credit spread in the new rating and the old rating. This is a downgrade from AA to A so spreads will widen out. Therefore, the return will be negative – wider spread leads to higher yield results in a lower price.

Expected percentage price change = $-4.2 \times (0.0175 - 0.01) = -0.0315$, or –3.15%.

The correct answer is C.

Practice Problem 7

7. Kreming's suggested model for Bond IV is a:

   A. structural model.

   B. reduced-form model.

   C. term structure model.

Practice Problem 7

A reduced-form model in credit risk analysis uses historical variables, such as financial ratios and macroeconomic variables, to estimate the default intensity. A structural model for credit risk analysis, in contrast, uses option pricing and relies on a traded market for the issuer’s equity.

The correct answer is B.

Practice Problem 8

8. Which of Kreming's observations regarding actual and risk-neutral default probabilities is correct?

   A. Only Observation 1

   B. Only Observation 2

   C. Both Observation 1 and Observation 2

Practice Problem 8

The actual default probabilities do not include the default risk premium associated with the uncertainty in the *timing* of the possible default loss. They are not that precise! Therefore Observation 1 is incorrect.

The observed spread over the yield on a risk-free bond in practice does include liquidity and tax considerations, in addition to credit risk. Observation 2 is correct

The correct answer is B.

Practice Problem 9

Which of the following statements regarding credit ratings is *least* accurate?

- **A)** A disadvantage of traditional credit ratings is that they are stable over time which reduces the correlation with a debt offering's default probability.
- **B)** An advantage of traditional credit ratings is that they provide a simple way of summarizing complex credit analysis.
- **C)** An advantage of traditional credit ratings is that they tend to vary with the business cycle which accurately reflects current risk.

Practice Problem 9

Traditional credit ratings do not fluctuate with the business cycle. They are designed to be long term in nature and often only change when there is significant new information pertaining to the issuer such as securities issuance, acquisition, etc.

The correct answer is C.

Practice Problem 10

Which of the following two securities are *most likely* used to calculate the term structure of credit spreads?

- **A)** A corporate issuer's senior debt and the same issuer's subordinated debt.
- **B)** A corporate issuer's coupon paying bond and the same issuer's zero coupon bond.
- **C)** A corporate issuer's zero coupon bond and a default free zero coupon bond.

Practice Problem 10

The implication here is for a generic credit spread which generally compares an issuer's senior debt against a risk-free benchmark. Term structure also implies a comparison to similar maturities, not bonds of the same issuer.

In order to be meaningful a spread must be off the same discount curve – either zero coupon or par rates.

The correct answer is C.

Practice Problem 11

An investor currently holds a zero coupon bond that matures in two years and has a face value of $100,000. The continuously compounded risk free rate is 0.60% an the bond issuer's credit spread is 0.25%. The present value of the expected loss implied by the credit spread is *closest* to:

- **A)** $1,679.
- **B)** $246.
- **C)** $493.

Practice Problem 11

Find the difference between the prices of the risk free and risky zero- coupon bonds.

Risk free yield	0.60%	
Credit spread	0.25%	
Total yield	0.85%	
Present value on a risk free basis:		$\$100{,}000 \times e^{-(0.0060 \times 2)} = \$98{,}807$
Present value on risky basis:		$\$100{,}000 \times e^{-(0.0085 \times 2)} = \$98{,}314$

Present value of expected loss due to credit risk = \$98,807 − \$98,314 = \$493

The correct answer is C.

Trading Terminology

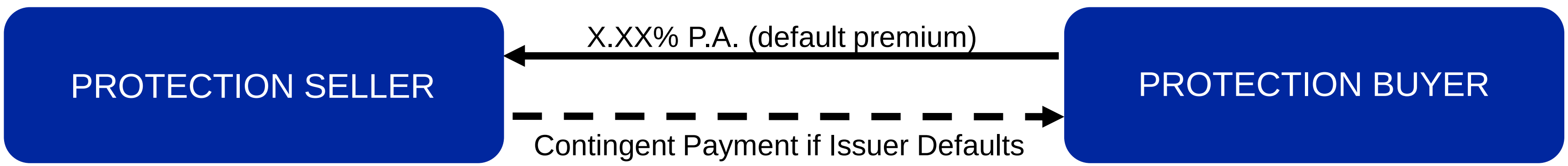
Bond and derivatives market trading terminology:

Buyers want the underlying instrument to go up in price (lower in yield and/or spread) while sellers want it to go down (higher in yield and/or spread).

Market	Buyer/Owner	Seller/Short	Note
Cash bonds	Buyer	Seller	
Futures and Options	Long	Short	"Longs take delivery, shorts make delivery"
Interest Rate Swaps	Receiver	Payer	Refers to the fixed coupon leg of the swap
Credit Default Swaps	Protection Seller	Protection Buyer	Selling protection is akin to selling insurance

Describe credit default swaps (CDS)

- In a Credit Default Swap the "Protection Buyer" pays a premium for the right to put bonds at par to the "Protection Seller" in case of a defined "default event".
- Premiums paid quarterly generally on the March, June, September, December cycle.
- Maturity (tenor) generally ranges from 1 to 10 years with 5 years being the most common.



- Receives Premium
- Is exposed to the issuer's credit
- Maximum loss is Default Price - Par
- Risk is the same as owning a floating rate bond of the issuer

- Pays Premium
- Is protected from exposure to the issuer's credit
- Can exercise put in case of specific default events:
  - Bankruptcy
  - Failure to pay
  - Restructuring

Credit default swaps (CDS) terminology

- Reference Entity: The actual borrower that the CDS is written on.
  - Can be very specific in terms of holding vs. operating company.
  - Sovereigns can be referenced as well.
- Reference Obligation: A specific debt instrument that is being covered.
  - Usually, a senior unsecured bond is referenced.
  - All other debt that is pari passu with this obligation is covered as well.
- Cheapest-to-Deliver: Which of the potentially deliverable outstanding bonds can be purchased and delivered into the CDS (if required) at the lowest cost.
- Notional amount: The size of the CDS contract.
  - Does not have to be related to and is not limited by either the size of the Reference Obligation or the Reference Entity's total debt outstanding.

Credit events

Credit Events – i.e. what triggers a CDS to pay out:

- Bankruptcy: When the Reference Entity actually files the legal paperwork.
- Failure to Pay: Missing a coupon or other significant cash flow. CDS does not actually trigger until the expiration of a grace period.
- Restructuring: A broad area which covers a number of scenarios. Has to be involuntary and forced upon creditors. May include a forced exchange of bonds with reduced and/or deferred principal and interest.

Succession Event – does not trigger a CDS pay out:

- Change in corporate structure such as merger, divestiture, spinoff or any similar even where the responsibility for the debt becomes unclear.

Settlement upon determination that a Credit Event has occurred:

- Parties have the right, but not the obligation, to settle. Settlement typically takes place 30 days after the declaration of a credit event and can be physical (bonds delivered) or cash. Actual payout determined by an auction of

Practice Problems 1 - 7

John Smith, a fixed-income portfolio manager at a €10 billion sovereign wealth fund (the Fund), meets with Sofia Chan, a derivatives strategist with Shire Gate Securities (SGS), to discuss investment opportunities for the Fund. Chan notes that SGS adheres to ISDA (International Swaps and Derivatives Association) protocols for credit default swap (CDS) transactions and that any contract must conform to ISDA specifications. Before the Fund can engage in trading CDS products with SGS, the Fund must satisfy compliance requirements.

Smith explains to Chan that fixed-income derivatives strategies are being contemplated for both hedging and trading purposes. Given the size and diversified nature of the Fund, Smith asks Chan to recommend a type of CDS that would allow the Fund to simultaneously fully hedge multiple fixed-income exposures.

Smith and Chan discuss opportunities to add trading profits to the Fund. Smith asks Chan to determine the probability of default associated with a five-year investment-grade bond issued by Orion Industrial. Selected data on the Orion Industrial bond are in Exhibit 1 below.

Year	Hazard Rate
1	0.22%
2	0.35%
3	0.50%
4	0.65%
5	0.80%

Practice Problems 1 - 7

Chan explains that a single-name CDS can also be used to add profit to the Fund over time. Chan describes a hypothetical trade in which the Fund sells £6 million of five-year CDS protection on Orion, where the CDS contract has a duration of 3.9 years. Chan assumes that the Fund closes the position six months later, after Orion's credit spread narrowed from 150 bps to 100 bps.

Chan discusses the mechanics of a long/short trade. In order to structure a number of potential trades, Chan and Smith exchange their respective views on individual companies and global economies. Chan and Smith agree on the following outlooks.

Outlook 1: The European economy will weaken.
Outlook 2: The US economy will strengthen relative to that of Canada.
Outlook 3: The credit quality of electric car manufacturers will improve relative to that of traditional car manufacturers.

Chan believes US macroeconomic data are improving and that the general economy will strengthen in the short term. Chan suggests that a curve trade could be used by the Fund to capitalize on her short-term view of a steepening of the US credit curve.

Another short-term trading opportunity that Smith and Chan discuss involves the merger and acquisition market. SGS believes that Delta Corporation may make an unsolicited bid at a premium to the market price for all of the publicly traded shares of Zega, Inc. Zega's market capitalization and capital structure are comparable to Delta's; both firms are highly levered. It is anticipated that Delta will issue new equity along with 5- and 10-year senior unsecured debt to fund the acquisition, which will significantly increase its debt ratio.

Practice Problem 1

1. To satisfy the compliance requirements referenced by Chan, the Fund is most likely required to:

A. set a notional amount.

B. post an upfront payment.

C. sign an ISDA master agreement.

Practice Problem 1

Parties to CDS contracts generally agree that their contracts will conform to ISDA specifications. These terms are specified in the ISDA master agreement, which the parties to a CDS sign before any transactions are made.

Therefore, to satisfy the compliance requirements referenced by Chan, the sovereign wealth fund must sign an ISDA master agreement with SGS.

The correct answer is C.

Practice Problem 2

2. Which type of CDS should Chan recommend to Smith?

A. CDS index

B. Tranche CDS

C. Single-name CDS

Practice Problem 2

“Smith asks Chan to recommend a type of CDS that would allow the Fund to simultaneously fully hedge multiple fixed-income exposures.”

A CDS index (e.g., CDX and iTraxx) would allow the Fund to simultaneously fully hedge multiple fixed-income exposures. A tranche CDS will also hedge multiple exposures, but it would only partially hedge those exposures.

The correct answer is A.

Practice Problem 3

3. Based on Exhibit 1, the probability of Orion defaulting on the bond during the first three years is closest to:

A. 1.07%.

B. 2.50%.

C. 3.85%.

Practice Problem 3

Based on Exhibit 1, the probability of survival for the first year is 99.78% (100% - 0.22% hazard rate). Similarly, the probability of survival for the second and third years is 99.65% (100% - 0.35% hazard rate) and 99.50% (100% - 0.50% hazard rate), respectively.

Therefore, the probability of survival of the Orion bond through the first three years is cumulative, calculated as $0.9978 \times 0.9965 \times 0.9950 = 0.9893$, and the probability of default sometime during the first three years is 1 – 0.9893, or 1.07%.

The correct answer is A.

Practice Problem 4

4. To close the position on the hypothetical Orion trade, the Fund:

A. sells protection at a higher premium than it paid at the start of the trade.

B. buys protection at a lower premium than it received at the start of the trade.

C. buys protection at a higher premium than it received at the start of the trade.

Practice Problem 4

The trade assumes that £6 million of five-year CDS protection on Orion is initially sold, so the Fund received the premium for providing default insurance.

Because the credit spread of the Orion CDS narrowed from 150 bps to 100 bps, Orion is now viewed as less risky and premium will be lower. The CDS position is marked at a gain. This gain is equal to the difference between the upfront premium received on the original CDS position and the upfront premium to be paid based on today's spread.

To close the position and monetize this gain, the Fund should unwind the position by buying protection for a lower premium.

The correct answer is B.

Practice Problem 5

5. The hypothetical Orion trade generated an approximate:

A. loss of £117,000.

B. gain of £117,000.

C. gain of £234,000.

Practice Problem 5

The Fund gains because it sold protection at a spread of 150 bps and closed out the position by buying protection at a lower spread of 100 bps.

The gain is calculated as:

~Profit = Change in credit spread (bps) x Duration x Notional Amount
= (150 – 100) x 3.9 x £6mm
= .005 x 3.9 x £6mm (remember 1bp = .0001)
= £ 117,000

The correct answer is B.

Practice Problem 6

6. Based on the three economic outlook statements, a profitable long/short tradewould be to:

A. sell protection using a Canadian CDX IG and buy protection using a US CDX IG.

B. buy protection using an iTraxx Crossover and sell protection using an iTraxx Main.

C. buy protection using an electric car CDS and sell protection using a traditional car CDS.

Practice Problem 6

Based on Outlook 1, Chan and Smith anticipate that Europe's economy will weaken. This implies that higher credit quality issuers will outperform lower credit quality issuers who are at higher risk of default. The trade is based on overall credit performance, not a specific name or industry.

In order to profit from this forecast, one would buy protection using a high-yield CDS index (e.g., iTraxx Crossover) and sell protection using an investment-grade CDS index (e.g., iTraxx Main).

The correct answer is B.

Practice Problem 7

7. The curve trade that would best capitalize on Chan's view of the US credit curve is to:

A. buy protection using a 20-year CDX and buy protection using a 2-year CDX.

B. buy protection using a 20-year CDX and sell protection using a 2-year CDX.

C. sell protection using a 20-year CDX and buy protection using a 2-year CDX.

Practice Problem 7

To take advantage of Chan's view of the US credit curve steepening in the short term, we want a position that will benefit if the difference between longer maturity spreads and shorter maturity spreads increases.

This could entail buying protection on a long-term (20-year) CDX and selling protection on a short-term (2-year) CDX.

The correct answer is B.

Practice Problem 8

8. A profitable equity-versus-credit trade involving Delta and Zega is to:

A. short Zega shares and buy protection on Delta using the 10-year CDS.

B. go long Zega shares and buy protection on Delta using 5-year CDS.

C. go long Delta shares and buy protection on Delta using 5-year CDS.

Practice Problem 8

If Delta bids for Zega the shares of Zega should rise. We want to own Zega's equity.

If Delta issues significantly more debt to finance the acquisition, then Delta becomes more leveraged which makes it riskier, subject to a downgrade, etc. Delta's credit spread is likely to widen as a result.

An equity-versus-credit trade would be to buy the Zega shares and buy protection on Delta.

*** Note the phrase in the reading "If the Fund sells protection on Delta now, the trade will realize a profit as credit spreads widen." is completely wrong.

The correct answer is B.