

CFALA/USC REVIEW MATERIALS USING THE TI-BAII PLUS CALCULATOR

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(helpful if you put CFA Review in subject line)

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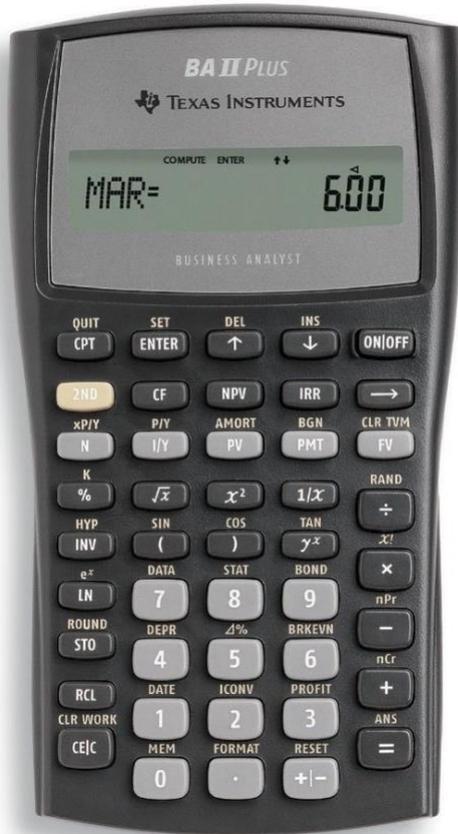
**CFA Society
Los Angeles**

Using the TI-BA2+

Notes by David Cary

- These should help you be more efficient with your calculator.
- Part I is basic settings for the calculators, and
- Part II is showing how to use the built-in functions to solve problems on the exam.
- For some of the topics I show more than one way to do a calculation.
- Some of these slides are in the Level I lecture notes and others are new.

Part I: CFA Exam acceptable calculators



TI BA II Plus
(original)



TI BA II Plus
Professional



TI BA II Plus
Professional (new)

While the original may still be available for purchase, it is missing some functions the new versions have and the battery is harder to replace

Have a fresh battery, consider taking a spare

CFA Exam acceptable calculators: *Texas Instruments*



- “Calculator covers, keystroke cards, and loose batteries are permitted in the testing room; instruction manuals are not. You may keep a small screwdriver with you if necessary to replace batteries in the BA II Plus.” (from CFAI website)
- **DC:** Switch from Chn to AOS by [2nd][FORMAT][↑], shows Chn or AOS. If Chn, [2nd] [SET] to convert to AOS. AOS uses the rules of Algebra for equations!

PART I: BASIC SETUP

- To set the number of decimal places:

Keys	Display	Comments
[2 nd][FORMAT]	DEC = 2.00	Shows current setting
4 [ENTER]	DEC= 4.0000	New setting
6 [ENTER]	DEC= 6.000000	New setting, etc.
CE/C	6.000000	Ready to go

I usually set for 4 places then change if necessary.

To clear memories use for specific types of calculations

→	Deletes last digit entered
[CE/C]	Sets display value to 0
[2 nd] [CLR TVM] (<i>above FV key</i>)	Sets N, I, PV, PMT, FV to 0.0
[2 nd] [DATA] [2 nd] [CLR WORK]	Clears Statistical memories
[CF] [2 nd] [CLR WORK]	Clears Cash Flow memories

PART I: ORDER OF OPERATIONS

- In the rules of algebra, calculations should be done in the following order:
 - Items in parentheses, exponents and roots, multiplication and division, and finally addition and subtraction.
 - For items in equal levels, go from left to right.
- For example, the correct calculation order for “ $2 \times 3 + 4 \times 5$ ” should be $2 \times 3 = 6$, “ $4 \times 5 = 20$ ” and then “ $6 + 20 = 26$ ”.
- BUT, the default method in the TI calculator (Chn) will probably chain the calculations in left to right order and do “ $2 \times 3 = 6$ ”, “ $6 + 4 = 10$ ”, “ $10 \times 5 = 50$ ”. Not the right answer.
- There are three methods discussed below to get the correct answer, relatively efficiently, with examples:
 1. Using your calculator memories,
 2. Using parentheses, and
 3. Switching the TI-BAlI+ into “AOS” “Algebraic Operating System.”

PART I: USING MEMORIES OR PARENTHESES:

Using Memories:

- You could do 2 [x] 3 [=] 6 and store in Memory 1: [STO][1], and then
- Do 4 [x] 5 [=] 20 and either
- [+] [RCL][1] [=] “26”, or
- [STO] [2], then [RCL][1] + [RCL][2] = “26”

Using parentheses:

- 2 [x] 3 [+] [2nd][(] 4 [x] 5 [2nd [)] [=] “26”

PART I: USING AOS

To get to AOS, use [2nd] [FORMAT] and the screen will show the decimal setting.

- Use up arrow and it will say Chn or AOS.
- If it says Chn, use [2nd] [SET] to change to AOS.
- If it says AOS, it is ready to go.
- You can use [2nd] [SET] again to change back to Chn.
- Then hit [CE/C] to start.
- *(You can switch back and forth if you wish, I stay with AOS!)*

The Algebraic Operating System follows the rule of order for algebra.

- For the same example it would do the following:
- When you enter the '+' sign after 2x3 it multiplies the 2x3 to get 6 as multiplication has priority over addition.
- It holds the "6+" in memory to see what is next.
- When the 4 is entered it doesn't do anything yet.
- Then when the "x 5 =" is entered it knows it should multiply "4x5" and then add "6" to get the correct answer of "26".

WHICH METHOD TO USE: AOS OR CHN?

- I like AOS. For the simpler calculations you can do them directly. For more complicated calculations you can still use the memory or parenthesis methods even if the calculator is in AOS mode.
- If you want to use AOS mode, please check on a regular basis it is still in AOS mode. An accidental reset may change it back to Chn.
- Not sure why you would want to but you can easily change back and forth between AOS and Chn by using the [2nd][FORMAT], then up arrow, then [2nd][SET].

PART I: USING MEMORIES

- Your calculator has 10 easily accessible memories: #0 to #9.
- You can store numbers in a memory by pressing [STO] [n] where [n] is the number of the memory where you want to store the number.

For example: 3 [STO] [1] will store the number 3 in memory #1.

- You can recall a stored number by pressing [RCL] [n] where [n] is the number of the memory you want to recall. For example: [RCL] [1] will give the value 3 (assuming you stored it in the previous step above).
- You do not need to clear memories because when you store a number it writes over any value that may have been there before.

PART I: ADDING, SUBTRACTING, MULTIPLYING AND DIVIDING TO MEMORIES!

- The following steps can also be done with memories, but be careful, one missed step messes up the numbers.
 - 3 [STO][1] puts 3 in Memory #1
(assume each step below is done in sequence).
 - Then if you press 4 [STO] [+] [1], the value in #1 will be 7.00,
($3+4=7$)
 - Then if you press 4 [STO] [-] [1], the value in #1 will be 3.00,
($7-4=3$)
 - Then if you press 4 [STO] [x] [1], the value in #1 will be 12.00,
($3 \times 4=12$) and
 - Then if you press 4 [STO][÷] [1], the value in #1 will be 3.00
($12 \div 4=3$).

TI: Y^X FUNCTION:

Assume you want to calculate $100 [\times] 1.05^3 = 115.76$. This is calculating the future value of putting 100 in the bank for 3 years at 5%:

- **This is for the Chn mode.** I PREFER THE ALG MODE!

The TI calculator has a Y^X function, but it must be used carefully, especially in Chn mode. If you multiply 100 [x] 1.05 before raising to the 3rd power, the calculator will raise 105 to the 3rd power and give the wrong answer.

1. Do the “1.05 [Y^X] 3” first and then multiply by 100, or
 2. Do “100 [×] [(]1.05 [Y^X] 3 [=]”. The left parenthesis [(] will allow the calculator to do the 1.05^3 before multiplying by 100. You can add [)] after the “3” if you want.
- **In ALG mode**, just do: 100 [×] 1.05 [Y^X] 3 [=] “115.76”.
 - **Or use TVM function:** 3 [N], 5 [i], 100 [PV], [CPT] [FV] “= -115.76”.
Negative at end as you can take that amount out.

TI: Y^X FUNCTION:

Assume you want to calculate $100 \div 1.05^3 = 86.38$. This would be the present value of 100 in 3 years at 5% interest.

• **For Chn mode.** You have to

1. Do 1.05 [Y^X] 3 [=] “1.1576”, [1/x] “0.8638”, [×] 100 [=] “86.38”, or
2. 1.05 [Y^X] 3 [+/-] [=] “0.8638”, [×] 100 [=] “86.38”

• **For ALG mode:**

1. Do: 100 [÷] 1.05 [Y^X] 3 [=] “86.38”, or
2. Do the “1.05 [Y^X] 3” first, then [1/x] and then multiply by 100.

• **Or use TVM function:**

3 [N], 5 [i], 100 [FV], [CPT] [PV] “= -86.38”. Negative as you want to take out 100 at the end so you have to put 86.38 in at the beginning.

PART I: SPECIAL KEYS

Keys	Function	What it does
[+ -]		Changes value on display to negative or positive.
[2 nd] [=]	[ANS]	Brings back the last calculated value.
[2 nd] [x]	[x!]	Calculates X Factorial ($X \times (X-1) \times \dots \times 1$)
[2 nd] [-]	[nPr]	Calculates permutations for r items out of n
[2 nd] [+]	[nCr]	Calculates combinations for r items out of n
[2 nd] [0]	[MEM]	Use the down arrow[↓] to check memories
[2 nd] [2]	[ICONV]	Converts stated annual rate to effective annual rate or back (more details later in this slide pack)
[2 nd] [.]	[FORMAT]	DEC Set number of decimal places, enter number of places and then hit [ENTER] key. If you hit '9' the number of decimal places varies with the answer.
[2 nd] [.] [↑]	[Chn] or [AOS]	[Chn] does calculation in order entered. [AOS] uses rules of Algebra (paranatheses, powers, multiplication and division, addition and subtraction,.. Hit [2 nd][ENTER] “[SET]” to change.

PART I: SPECIAL KEYS

Keys	Function	What it does
$[\sqrt{x}]$	Sq Root	Calculates the square root of number in the display
16 $[\sqrt{x}]$	“4”	... square root of 4
$[x^2]$	Square	Calculates the square of the number in the display
4 $[x^2]$	“16”	... 4 squared
$[1/x]$	Reciprocal	Calculate the reciprocal of the number in the display
4 $[1/x]$	“0.25”	... reciprocal of 4
$[y^x]$	Power or Root	Raises the number in the display, y, to the ‘x’ power. If x is a fraction, to the ‘x’ root. See slides about y^x
4 $[y^x]$ 3 [=]	“64”	... cubes 4
4 $[y^x]$ 3 $[1/x]$ [=]	“1.5874”	Cube root of 4
4 [LN]	“1.3863”	Natural log of 4
4 $[2^{nd}]$ [LN] $[e^x]$	“54.5982”	Raises “e” to the 4 th power.

Calculator Hints – BEG mode

This is for Annuities Due where the payment is at the beginning of each period:

Enter	Shows	Explanation
[2 nd] [BEG]	END	Shows current setting
[2 nd] [SET]	BEG	Shows new setting
[CE/C]	^{BEG} 0.0000	Should show BEG above number
<i>To Reset after calculation...</i>		
[2 nd] [BEG]	BEG	Shows current setting
[2 nd] [SET]	END	Shows new setting
[CE/C]	0.0000	Blank above number

Note: The [BEG] key is above the [PMT] key.

CALCULATOR HINTS: # OF PMTS/YR

While I prefer to adjust N and I directly, the TI can be set to do the calculations (for daily or ...):

Enter	Shows	Explanation
[2 nd] [I/Y]	P/Y = 1.0000	Shows current setting
365[ENTER]	P/Y = 365.0000	Shows new setting
[CE/C]	365.0000	P/Y set
<i>To Reset after calculation...</i>		
[2 nd] [I/Y]	P/Y = 365.0000	Shows current setting
1[ENTER]	P/Y = 1.0000	Shows new setting
[CE/C]	1.0000	P/Y set

When you use P/Y, enter $N = (P/Y) \times (\# \text{ of years})$, and $I = I$.
I would recommend getting into the habit of checking the P/Y setting before doing TVM problems

Using the TI-BA II+ for effective rate

Calculate the effective annual rate if the stated (nominal) rate is 18%, compounded monthly.



Steps	Screen	Comment
[2 nd] [2]	NOM= 0.000000	Starts the process
18 [ENTER]	NOM= 18.000000	Enters nominal rate
↓	EFF= 0.000000	To be calculated
↓	C/Y= 1.000000	Compound periods per year
12 [ENTER]	C/Y= 12.000000	Enters number of compounding periods per year
↓	NOM= 18.000000	Shows nominal rate again
↓	EFF= 0.000000	To be calculated
[CPT]	EFF= 19.561817	THE ANSWER!

Note, if given the effective annual rate, you can enter it in 'EFF' and then calculate the nominal [NOM] rate and vice versa.

Using the TI-BA II+ for effective rate

- You can “cheat” with this function to calculate continuously compounded rates. Just set C/Y to 100,000.
- Calculate the effective rate if 18.0% per year is continuously compounded. $EAR = e^{0.180} - 1 = 19.7217\%$

Steps	Screen	Comment
[2 nd] [2]	NOM= 0.0000	Starts the process
18 [ENTER]	NOM= 18.0000	Enters nominal rate
↓	EFF= 0.0000	To be calculated
↓	C/Y= 1.0000	Compound periods per year
100,000 [ENTER]	C/Y= 100,000.0000	Enters number of compounding periods per year
↓	NOM= 18.0000	Shows nominal rate again
↓	EFF= 0.0000	To be calculated
[CPT]	EFF= 19.7217	THE ANSWER!

Reading 6, LOS c

Using the TI-BA II+ for effective rate

- You can “cheat” with this function to calculate continuously compounded rates. Just set C/Y to 100,000.
- Calculate the stated rate if the continuously compounded effective rate is 18.0% per year is. $SAR = LN(1.18) = 16.5514\%$

Steps	Screen	Comment
[2 nd] [2]	NOM= 0.0000	Starts the process
↓	EFF= 0.0000	Down to Effective Rate
18 [ENTER]	EFF = 18.0000	Enter Effective rate
↓	C/Y= 1.0000	Compound periods per year
100,000 [ENTER]	C/Y= 100,000.0000	Enters number of compounding periods per year
↓	NOM= 0.0000	Shows nominal rate again
[CPT]	NOM = 16.5514	THE ANSWER!

Reading 6, LOS c

Part II:

Before Starting TVM Calculations:

This is to calculate the Present Value (PV), Future Value (FV), Annuity Payment (PMT), Number of periods (N), and/or Interest rate (I).

Clear TVM memories

- TI: [2nd] [CLR TVM]



Entering TVM values

If you invest 1,000 for 5 years at 8%, how much will you have?

Find the FV if $N = 5$, $I = 8$, $PV = 1,000$

- Enter the value and then hit the key:
- 5 [N], 8 [I], 1000 [PV]

To get the answer:

- [CPT] [FV] “-1,469.33” *note, if you enter PV as positive, FV will be negative and vice versa.*
- 5 [N], 8 [I], 1000[+/-] [PV], [CPT] [FV] “1,469.33”

Entering TVM values

Note: if solving for number of periods or the interest rate, one or more value(s) (PV, PMT, FV) need to be entered as negative.

- Put 100 in the bank, grows to 200 in 8 years, calculate implied interest rate (make either the 100 or 200 negative, not both):
- 8[N], 100[+|-][PV], 200[FV], [CPT][I/Y] “*I/Y = 9.05*”

If you put 100 in the bank and add 20 at the end of each year for 8 years and it grows to 400, calculate the implied interest rate (the PV and PMT should have the same sign, the FV the opposite):

- 8 [N], 100[+|-][PV], 20[+|-][PMT], 400[FV], [CPT][I/Y] “*I/Y = 8.10*”
- Or 8 [N], 100 [PV], 20 [PMT], 400 [+|-] [FV], [CPT][I/Y] “*I/Y = 8.10*”

...or a series of unequal cash flows;

PV or FV of Uneven Cash Flows

- Table Method: calculate the PV (or FV) of each cash flow and add them together.
- Or learn how to use 'CF' keys on your calculator and save TIME!
- Example: Find PV of receiving \$100 at the end of year 1, \$200 at the end of year 2, \$400 at the end of year 3 and \$600 at the end of year 4, using 10%, and then add all those up:

$$100/1.1 + 200/1.1^2 + 400/1.1^3 + 600/1.1^4 = ???$$

PV OF UNEVEN CASHFLOWS

$$CF_0 = 0, CF_1 = 100, CF_2 = 200$$

$$CF_3 = 400, CF_4 = 600, I = 10\%$$

Key Strokes	Explanation	Display
[CF][2nd][CLR WORK]	Clear CF Registers	CF0 = 0.00000
0 [Enter]	Initial Outflow = 0	CF0 = 0.00000
 ↓ 100 [Enter]	Enter CF1	C01 = 100.00
↓↓ 200 [Enter]	Enter CF2	C02 = 200.00
↓↓ 400 [Enter]	Enter CF3	C03 = 400.00
↓↓ 600 [Enter]	Enter CF4	C04 = 600.00
[NPV] 10 [Enter]	Enters Interest Rate	I = 10.000
↓ [CPT]	Calculate NPV	NPV = 966.53
	(PV in this case)	

To get FV, [CE/C] (*display = 966.53234*)
 [PV] (*enters 966.53234 as PV*)
 10 [I/Y] (*enters 10% interest*)
 4 [N] (*enters 4 years*)
 [CPT] [FV] (*Display = -1,415.10*) This is actually a positive value, if PV is positive, FV is given as negative

With TI-BAII+Pro
 After NPV, ↓,
 NFV, [CPT],
 NFV = 1,415.10

NPV, IRR EXAMPLE

- Assume a project costs \$1,000. It will generate cash flows of \$100, \$200, \$400, \$600 for the next 4 years¹. The discount rate is 10%. Calculate NPV and IRR.
- $CF_0 = -1000$, $CF_1 = 100$, $CF_2 = 200$,
 $CF_3 = 400$, $CF_4 = 600$, $I = 10\%$
- $NPV = -33.47$
- $IRR = 8.79\%$, note $NPV < 0$, $IRR < \text{discount rate}$.
- Detailed steps are on next slide.

¹ Like the previous example, except for initial cost.

NPV & IRR OF UNEVEN CASH FLOWS

$I = 10\%$, $CF_0 = -1000$, $CF_1 = 100$,
 $CF_2 = 200$, $CF_3 = 400$, $CF_4 = 600$

Key Strokes	Explanation	Display
[CF] [2nd] [CLR WORK]	Clear CF Registers	CF0 = 0.00
1000 [+/-] [Enter]	Initial Outflow	CF0 = -1000.00
 ↓ 100 [Enter]	Enter CF1	C01 = 100.00
↓↓ 200 [Enter]	Enter CF2	C02 = 200.00
↓↓ 400 [Enter]	Enter CF3	C03 = 400.00
↓↓ 600 [Enter]	Enter CF4	C04 = 600.00
[NPV] 10 [Enter]	Enters Interest Rate	I = 10.00
↓ [CPT]	Calculate NPV	NPV = -33.47
[IRR] [CPT]	Calculate IRR	IRR = 8.79

To get FV, [CE/C] (*display = -33.47*)
 [PV] (*enters -33.47 as PV*)
 10 [I/Y] (*enters 10% interest*)
 4 [N] (*enters 4 years*)
 [CPT] [FV] (*Display = 49.00*) *This is actually a negative FV*

With TI-BAII+Pro
 After NPV, ↓,
 NFV, [CPT],
 NFV = -49.00

Viewing and/or Correcting Cash Flow for inputs for NPV and IRR

After inputting the Cash Flows for an NPV or IRR calculation, you can see the values by going back to the [CF] key and use the arrows. For example,

[CF]	Shows CF ₀
[↓]	Shows CF ₁
[↓]	Shows F ₁
[↓]	Shows CF ₂
Etc...	

F₁ is used if the cash flow is repeated several years in a row.

You can correct a CF input when you find an error by using the enter key. For example, assume you entered -10 for CF₀ when it should have been -1000.

Key	Shows
[CF]	CF ₀ = -10
1000[+ -][ENTER]	CF ₀ = -1000

You can use the down arrows to find and correct any other cash flow errors.

Cash Flows

Note: on the exam they may try a trick:

- Assume the cash flows are
 $CF_0 = -1000$, $CF_1 = 100$, $CF_2 = 200$, $CF_3 = 0$,
 $CF_4 = 400$, $CF_5 = 500$.
- You **MUST** enter CF_3 as 0 or the remaining cash flows will be for the wrong periods and you will get the wrong answer!
- They could also make CF_1 negative!

Statistics: Practice Problem

Year	S&P
2007	13.5%
2008	-1.2%
2009	-35.6%
2010	32.4%
2011	16.5%
2012	3.8%
R_f	3.0%

Calculate the following:

1. Mean
2. Standard Deviation
3. Coefficient of Variation
4. Sharpe Ratio

Similar to example in D.Cary lecture notes.

TI-BAII+ (and Pro):	On Screen	
[2 ND][Data]	X01 0.0000	Opens Input
[2 ND][CLR WORK]	X01 0.0000	Clears Memories
13.5 [ENTER]	X01 = 13.5000	Enters 1 st value
↓↓1.2 [+/-][ENTER]	X02 = -1.2000	Enters 2 nd value
↓↓35.6 [+/-][ENTER]	X03 = -35.6000	Enters 3 rd value
↓↓32.4 [ENTER]	X04 = 32.4000	Enters 4 th value
↓↓16.5 [ENTER]	X05 = 16.5000	Enters 5 th value
↓↓3.8 [ENTER]	X06 = 3.8000	Enters 6 th value
[2 ND] [STAT]	LIN	
↓	N = 6.0000	Number of items
↓	\bar{x} = 4.9000	Mean
↓	S _x = 22.9932	Standard Deviation

You can view your inputted values by going into DATA mode and then using up or down arrows.

You use the two down arrows as the calculator can use x & y values for a regression.

Practice Problem continued...

$$\text{Mean} = r = 4.90\%$$

$$\text{StDev} = s = 22.99\%$$

$$3. \text{ Coefficient of Variation} = s / r$$

$$\text{CV} = 22.99\% / 4.90\%$$

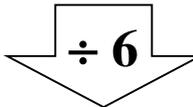
$$= 4.69$$

$$4. \text{ Sharpe} = (r - r_f) / s$$

$$= (4.90\% - 3.0\%) / 22.99\%$$

$$= 0.083$$

If asked for the Mean Absolute Deviation: No shortcut!

Year	S&P	S&P - Mean
2007	13.50%	8.60%
2008	-1.20%	6.10%
2009	-35.60%	40.50%
2010	32.40%	27.50%
2011	16.50%	11.60%
2012	<u>3.80%</u>	<u>1.10%</u>
sum	29.40%	95.4%
Mean	4.90%	 ÷ 6
median	8.65%	
MAD		15.90%

Geometric and Harmonic Mean

Using the Statistics Function for the calculation is after this slide.

- Data: 3, 3, 4, 6, 22

Geometric Mean (ALG)	
KEYS	DISPLAY
3 [×]	3.0000
3 [×]	9.0000
4 [×]	36.0000
6 [×]	216.000
22 [=]	4,752.0000
[y ^x] 5 [1/x] [=]	5.4372

Harmonic Mean (ALG)	
KEYS	DISPLAY
3 [1/x] [+]	0.3333
3 [1/x] [+]	0.6667
4 [1/x] [+]	0.9167
6 [1/x] [+]	1.0833
22 [1/x] [=]	1.1288
[÷] 5 [=]	0.2258
[1/x]	4.4295

From D.Cary's notes, SS 2, for "STATISTICAL CONCEPTS AND MARKET RETURNS".

- The next two slides show how to use the built-in statistic functions to do Geometric and Harmonic Average calculations.
- I'm not sure they are any easier, but thought I would include them so you can compare.

Using the Statistics Function to Calculate Geometric Mean

Note: converting the numbers to logs, calculating the average, then using e^x is the same as multiplying the numbers and taking the root.

- Data: 3, 3, 4, 6, 22

Keys	Display	Comments
[2 nd] [DATA] [2 nd] [CLR WORK]	X01 0.0000	Clears Memories
3 [LN] [ENTER]	X01=1.0986	Converts to Log, Enters first value
↓↓3 [LN] [ENTER]	X02=1.0986	Converts to Log, Enters second value
↓↓4 [LN] [ENTER]	X03=1.3863	Converts to Log, Enters third value
↓↓6 [LN] [ENTER]	X04=1.7918	Converts to Log, Enters fourth value
↓↓22 [LN] [ENTER]	X05=3.0910	Converts to Log, Enters fifth value
[2 nd] [STAT]	LIN	Just shows in linear format
↓	N = 5.0000	
↓	\bar{x} = 1.6933	The average of the logs
[2 nd] [e^x]	5.4372	Converts the average of the logs to the answer!

Probably easier to just multiply the numbers and take the root!

Using the Statistics Function to Calculate Harmonic Mean

• Data: 3, 3, 4, 6, 22

Keys	Display	Comments
[2 nd] [DATA] [2 nd] [CLR WORK]	X01 0.0000	Clears Memories
3 [1/x] [ENTER]	X01 = 0.3333	Converts to reciprocal, Enters value
↓ ↓ 3 [1/x] [ENTER]	X02 = 0.3333	Converts to reciprocal, Enters value
↓ ↓ 4 [1/x] [ENTER]	X03 = 0.2500	Converts to reciprocal, Enters value
↓ ↓ 6 [1/x] [ENTER]	X04 = 0.1667	Converts to reciprocal, Enters value
↓ ↓ 22 [1/x] [ENTER]	X05 = 0.0455	Converts to reciprocal, Enters value
[2 nd] [DATA]	LIN	Just shows in linear format
↓	N = 5.0000	Number of observations
↓	\bar{x} = 0.2258	Gets average of reciprocals
[1/x]	4.4295	<i>The reciprocal of the average reciprocal. The Answer!</i>

Viewing and/or Correcting for inputs when using STAT function

After inputting the values for a calculation, you can see the values by going back to the [2nd] [DATA] key and use the arrows. For example,

[2 nd] [DATA]	Shows XØ1
[↓]	Shows YØ1
[↓]	Shows XØ2
[↓]	Shows YØ2
Etc...	

“Y” values not used for most of level 1!

You can correct an input value when you find an error by using the enter key. For example, assume you entered 30 for XØ1 when it should have been 3.

Key	Shows
[2 nd] [DATA]	XØ1 = 30
3[ENTER]	XØ1 = 3
You can use the down arrows to find and correct any other errors.	

CALCULATING MEAN AND COVARIANCE OF TWO ASSET PORTFOLIO

	1	2
w	75%	25%
r	20%	12%
var	625	196
Covar	120	

- Example from D.Cary's Level I CFALA review lecture notes, SS 2, Reading 8:
- The question is to find the expected return " r_p " and the variance (and standard deviation) of this portfolio.
- Before doing the calculations, look at the numbers carefully.
 - A weight of 75% will be entered as 0.75
 - A return of 20% can be entered as 20 and the answer will be in "%" value or as 0.20 and the answer will be in decimal value "0.00".
 - Note if they give you Variance or Standard Deviation. For part of the equations used you might need variance and other parts might need standard deviation.
 - Also note if they give you the covariance or correlation of the two assets. Remember that covariance = correlation $\times s_1 \times s_2$.
 - The following pages use " s_{12} " to denote covariance.

CALCULATING MEAN AND COVARIANCE OF TWO ASSET PORTFOLIO

	1	2
w	75%	25%
r	20%	12%
var	625	196
Covar	120	

1. USING MEMORIES (Chn or AOS):

- $r_p = r_1 \times w_1 + r_2 \times w_2$
 1: 0.75 [×] 20 [=] '15' [STO] [1] “stores first step in #1”
 2: 0.25 [×] 12 [=] '3' [STO] [2] “stores second step in #2”
 3: [RCL] [1] + [RCL][2] = '18' “adds #1 and #2 to get answer.
- $s_p^2 = w_1^2 \times s_1^2 + w_2^2 \times s_2^2 + 2 \times w_1 \times w_2 \times s_{12}$
 1: 0.75[x²] [×] 625 [=] '351.56' [STO][1]
 2: 0.25[x²] [×] 196 [=] '12.25' [STO][2]
 3: 2 [×] 0.75 [×] 0.25 [×] 120 [=] '45.00' [STO] [3]
 4: [RCL] [1] [+] [RCL][2] [+] [RCL][3] [=] '408.81' “the variance”
 5: [√x] = 20.22% “the standard deviation is 20.22%”
- Remember if they give you the standard deviations, you have to square them to get the variance for the first two parts of the equation just like you squared the weights. If they give you correlation you have to multiply by the standard deviations to get covariance for the third part.

CALCULATING MEANS AND COVARIANCES OF TWO ASSET PORTFOLIO

	1	2
w	75%	25%
r	20%	12%
var	625	196
Covar	120	

2. USING PARENTHESES (Chn or AOS):

- $r_p = r_1 \times w_1 + r_2 \times w_2$
 1: 0.75 [×] 20 [+] [(]0.25 [×] 12[)] [=] '18'
“the parentheses are above the numbers 7 & 8”
“they are NOT needed for the first calculation since it is multiplication”
“you do not really need the last parenthesis, the ‘=’ will be enough”
- $s_p^2 = w_1^2 \times s_1^2 + w_2^2 \times s_2^2 + 2 \times w_1 \times w_2 \times s_{12}$
 1: 0.75[x²] [×] 625 [+] (0.25[x²] [×] 196) [+] [(]2 [×] 0.75 [×] 0.25 [×] 120[)]
 [=] '408.81' *“the variance”*
 5: [√x] = '20.22' *“the standard deviation in percent”*
- Even though the parentheses method works fine, I would consider using the memory method on the previous slide to avoid errors for longer calculations, such as for the variance.
- See previous bottom note if given standard deviations and/or correlations.

CALCULATING MEANS AND COVARIANCES OF TWO ASSET PORTFOLIO

	1	2
w	75%	25%
r	20%	12%
var	625	196
Covar	120	

3. USING AOS (NOT Chn):

To get to AOS, use [2nd] [FORMAT] and the screen will show the decimal setting.

Use up arrow and it may say Chn or AOS. If it says Chn, use [2nd] [SET] to change to AOS. If it says AOS, it is ready to go.

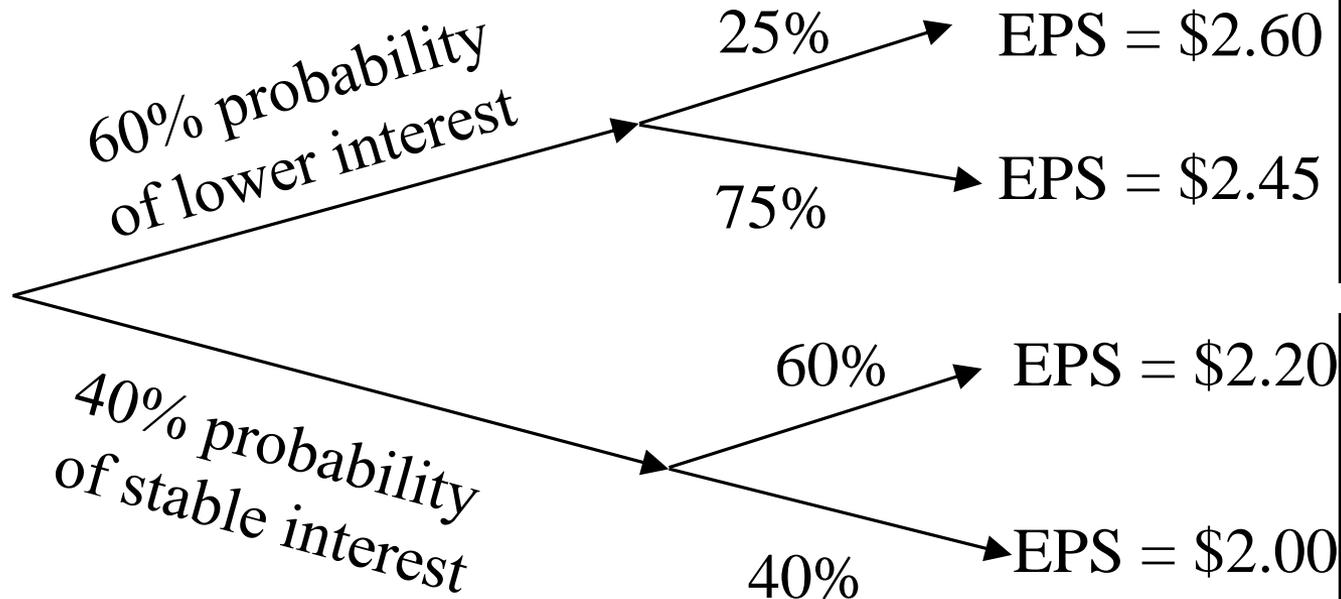
Then hit [CE/C] to start. (*You can switch back and forth, I use AOS*)

- $r_p = r_1 \times w_1 + r_2 \times w_2$
 1: 0.75 [×] 20 [+] 0.25 [×] 12 [=] '18'
“no parenthesis or memories needed for calculation!”
- $s_p^2 = w_1^2 \times s_1^2 + w_2^2 \times s_2^2 + 2 \times w_1 \times w_2 \times s_{12}$
 1: 0.75[x²] [×] 625 [+] 0.25[x²] [×] 196 [+] 2 [×] 0.75 [×] 0.25 [×] 120 [=]
 '408.81' *“the variance”*
 5: [√x] [=] '20.22' *“the standard deviation is 20.22 (in percentage)”*
- See previous bottom note if given standard deviations and/or correlations.

Tree Diagram

- Assume there is a 60% probability that interest rates will decrease and if they do, there is a 25% chance that $\text{EPS} = \$2.60$ and a 75% chance that $\text{EPS} = \$2.45$.
- Also, there is a 40% probability that interest rates will be stable and if so, there is a 60% probability that $\text{EPS} = \$2.20$ and a 40% probability that $\text{EPS} = \$2.00$
- *Note: Actual calculator steps included in following slides.*

Figure 8-2



Expected Value
if rates decrease:
 0.25×2.60
 $+ 0.75 \times 2.45$
 $= \$2.4875$

Expected Value
if stable rates:
 0.6×2.20
 $+ 0.4 \times 2.00$
 $= \$2.12$

- Overall Expected Value:
 $0.6 \times (0.25 \times 2.60 + 0.75 \times 2.45)$
 $+ 0.4 \times (0.6 \times 2.20 + 0.4 \times 2.00) = \2.34
- Note: $0.6 \times 2.4875 + 0.4 \times 2.12 = \2.34

DECISION TREES (USING MEMORIES)

Keys (Using Memories)	Display	Comments
$0.60 \times 0.25 \times 2.60 =$ [STO][1]	0.3900	Top Branch
$0.60 \times 0.75 \times 2.45 =$ [STO][2]	1.1025	2 nd Branch
$0.40 \times 0.60 \times 2.20 =$ [STO][3]	0.5280	3 rd Branch
$0.40 \times 0.40 \times 2.00 =$ [STO][4]	0.3200	Bottom Branch
[RCL][1] [+] [RCL][2] [+] [RCL][3] [+] [RCL][4] [=]	2.3405	Answer

Note: Quick check for reasonableness, values between 2.60 and 2.00, answer is about in the middle, that is reasonable!

- The next slide shows how to do this using the ‘1-V’ (1 Variable) function as part of the DATA/STAT functions. For this calculation I think using the memories above is probably faster and simpler.
- I include the “1-V” method both as an alternative and so you can see it being used.

You can also use the DATA/STAT function in the '1-V' mode, Since there are two probabilities for each branch, the "Y" values must be calculated with " $\% \times \text{decimal} = 60 \times 0.25 = 15$ " to get the right input.

Keys (Using STAT)	Display	
[2 nd][DATA] [2 nd][CLR WORK]	XØ1 = 0.0000	Clear Memories
2.60[ENTER]	XØ1 = 2.6000	Enters XØ1
↓60x0.25=[ENTER]	YØ1 = 15.0000	Combined Probability for #1
↓2.45[ENTER]	XØ2 = 2.4500	Enters XØ2
↓60x0.75=[ENTER]	YØ2 = 45.0000	Combined Probability for #2
↓2.20[ENTER]	XØ3 = 2.2000	Enters XØ3
↓40x0.60=[ENTER]	YØ3 = 24.0000	Combined Probability for #3
↓2.00[ENTER]	XØ4 = 2.0000	Enters XØ4
↓40x0.40=[ENTER]	YØ4 = 16.0000	Combined Probability for #4
[2 nd][STAT]	LIN	
<i>Repeat [2nd][SET] until 1-V appears</i>	1-V	Go into 1-V mode
↓	n = 100.0000	To represent 100%
↓	\bar{x} = 2.3405	The answer

Factorial, Combinations, and Permutations

- N Factorial (N!) is when you multiply
 $N \times (N-1) \times (N-2) \times \dots \times 2 \times 1 = N!$
- There is a built in function in your calculator to do this:
 $[2^{\text{nd}}][x] = [x!]$, To get 5!: $5 [2^{\text{nd}}][x!] = 120$
- This can be useful for combination calculations:

To select a subset of 3 items out of 5:

$${}_5C_3 = 5! / ((5-3!)(3!)) = 10$$

Your TI has a built in function for this: $[2^{\text{nd}}][+] = [{}_nC_r]$

$${}_5C_3 = 5 [2^{\text{nd}}][{}_nC_r] 3 [=] 10$$

Factorial, Combinations, and Permutations

- This can also be useful for permutation calculations where you select a subset and then rank the items in the subset:

To select and rank a subset of 3 items out of 5:

$${}_5P_3 = 5! / (5-3!) = 60$$

Your TI has a built in function for this: $[2^{\text{nd}}][-] = [{}_nP_r]$

$${}_5P_3 = 5 [2^{\text{nd}}][{}_nP_r] 3 [=] 60$$

Calculating Bernoulli Binomial Distribution

$$p(x) = \left[\frac{n!}{(n-x)! \times x!} \right] p^x (1-p)^{n-x} = {}_n C_x p^x (1-p)^{n-x}$$



Calculate the probability of getting 3 heads when tossing a fair coin 5 times:

$$[5! / (2! \times 3!)] \times 0.5^3 \times 0.5^2$$

You can use the ${}_n C_r$ function to calculate the first part:

- TI: AOS mode (highly recommended):
 $5 [2nd] [{}_n C_r] 3 [x] 0.5 [y^x] 3 [x] 0.5 [y^x] 2 [=]$ “0.3125”
 Basically just like the equation!
- TI: Chn mode (not recommended):
 $5 [2nd] [{}_n C_r] 3 [x] [(] 0.5 [y^x] 3 [x] [(] 0.5 [y^x] 2 [=]$ “0.3125”
 Have to add “(“ before each of the parts using $[y^x]$.
- Or just calculate each of the three parts and store in memories 1, 2, and 3, and then multiply the memories!

List of topics in Dave Cary lecture Notes

- Please note that slide numbers may change as lecture notes are updated, so the slide referenced may be a slide or two before or after the reference below:
- SS 2
- Two-Stage DDM Problem (actually SS 14) #98
- Mean Absolute Deviation #129
- Covariance & Joint Probability Function #179
- SS 3
- Macaulay's Duration (actually SS 16) #132